Substandard Structured Settlement Annuities – Best Estimate Mortality: Support for the Modified Log-Linear Declining (LLD) Method

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Executive Summary

The mortality assumption for Substandard Structured Settlement Annuities (SSSA) presents an interesting challenge, as it must be inferred from actual ages and "rated ages," which express the reduction in life expectancy due to an impairment. Three major basic methods have been used in academic studies: the Rated Age method, the Constant Extra Deaths method, and the Log-Linear Declining method. These basic methods assign annual mortality rates differently, but each preserves the life expectancy assigned at issue. Unfortunately, these basic methods fail to accurately predict future mortality on actual blocks of experience study data.

Actuaries have developed modifications of these methods to correct for their weaknesses. We examine three of them: Modified Rated Age, Modified Constant Extra Deaths, and Modified Log-Linear Declining. We fit each to actual data using a "training period" of early data. We then test that model on experience from the later period, the "test period."

We found that the best method for developing a mortality assumption from SSSA experience study data is the Modified Log-Linear Declining method. It had the closest fit of expected to actual at higher durations, which is important for pricing and valuation of in-force blocks. More qualitatively, the parameters and modifications of this method also seem to be the most intuitive, allowing for easier interpretation and explanation as well as modification for actuarial judgment.

Although we advocate for a common methodology, we do not advocate for a single common assumption. Instead, the Modified Log-Linear Declining method should be applied anew to each block, to reflect idiosyncrasies such as mix of ages, conditions, marketing focus, and sales era.

I. Substandard Structured Settlement Annuities

For a more thorough understanding of Structured Settlement Annuities (SSA), please refer to [1] in the References section below. A Substandard Structured Settlement Annuity (SSSA) is one written on an annuitant who has an "impairment" that makes their mortality higher than that of a "standard" life of the same age. This white paper is only concerned with SSSA and the estimation of their mortality.

Consider a prospective annuitant, age x, who has an impairment. A casualty insurer has become obligated to pay this individual a defined annuity in the future. A life insurer sells the casualty insurer an SSSA, agreeing to pay those future payments to the prospective annuitant in return for a premium. The premium is based on "substandard (impaired) mortality rates" that reflect the condition of this annuitant. These rates are based on an underwriter's opinion as to the "rated age," say $x + r$, of the individual. The rated age is meant to summarize the mortality effect of the impairment in a single number. The difference between the rated age and the actual age, $r > 0$, is referred to as the rate-up. Using i to represent "impaired" and s to represent "standard," we have equation (1):

$$\boldsymbol{e_x^i} = \sum_{t=1}^{\infty} {}_t\boldsymbol{p_x^i} \equiv \boldsymbol{e_{x+r}^s} = \sum_{t=1}^{\infty} {}_t\boldsymbol{p_{x+r}^s}, \tag{1}$$

where the sum on the right side is calculated from a given standard mortality table.

II. The challenge of SSSA mortality

The actuary must come up with a set of mortality rates that lead to the stated life expectancy, for which there are infinitely many solutions. How can we choose one and justify it?

Several reasonably simple methods have been proposed. We will study three of them briefly and see whether any of them are adequate for our purposes. If not, we will consider modifications of these models and identify the best method.

First, how can we evaluate these methods, so that we can choose the best one or ones? We want to use a method that fits the historical experience data and, more importantly, that fits the future experience. In order to be confident that a method will fit in the future, we will compare the performance of candidate methods across a number of SSSA blocks by fitting methods to "training periods" and then evaluating over "test periods" as described in Section VII below.

III. Three basic methods for inferring mortality rates from life expectancy

As a basis of developing basic methods for estimating mortality rates of impaired individuals, please refer to [2], which compares basic mortality models. Each of the three basic methods below assign annual mortality rates using the issue age and rated age. Although they assign mortality rates differently, all three basic methods maintain the Principle of Equivalent Life Expectancy, which is that the equation (1) holds. (Later, in Section VI below, this principle will no longer hold as the basic methods are calibrated to fit the actual experience data.)

A. RATED AGE (RA)

The Rated Age method stipulates that the life expectancy for the impaired life age x will be set equal to that of a standard life of the rated age $x + r$, provided by the underwriter. This is shown in equation (2):

$$\boldsymbol{q^i_{x+t} = q^s_{x+r+t}} \textbf{ for } \boldsymbol{t = 0, 1, 2, \ldots.} \tag{2}$$

This method is a rather simple method and is easy to compute, but the link between issue age and rated age results in the undesirable property that mortality hits the end of the table at an attained age r years lower than the end of the standard table.

B. CONSTANT EXTRA DEATHS (CED)

The Constant Extra Deaths method assigns an additive constant to each year's mortality rate, determined to maintain the Principle of Equivalent Life Expectancy. We have equation (3):

$$\boldsymbol{q^i_{x+t} = q^s_{x+t} + CED} \textbf{ for } \boldsymbol{t = 0, 1, 2, \ldots,} \tag{3}$$

and the additive constant is uniquely determined from equation (1). This method has the unavoidable property that the amount added to the mortality rate is large relative to the standard mortality rate in the early durations, and it is small in the later durations.

C. LOG-LINEAR DECLINING (LLD)

In this method, each impaired mortality rate is a multiple of the standard mortality rate for the same age. This multiple is called the Relative Risk, symbolized RR_t, as shown in equation (4):

$$\boldsymbol{q^i_{x+t} = RR_t \cdot q^s_{x+t}.} \tag{4}$$

We define the multiples to decrease exponentially toward 1 at a given future age, α, which is constant over each block of SSSA. This gradual decrease in the multiplier allows the impairment to wear off such that it does not overestimate mortality at higher durations. Also, α is a parameter that is adjusted for each specific block to fit the pattern of actual impairment wear-off. We say that the multiple decreases "toward 1" because the rate of change in the multiple may be slow enough that it doesn't reach 1 at an age within the mortality table. In a later section, we modify this method to allow for a quicker grade to standard mortality.

In symbols, for $x + t \leq \alpha$, we have equation (5):

$$\boldsymbol{RR_t = RR_0^{\left(\frac{\alpha - x - t}{\alpha - x}\right)}}, \tag{5}$$

and equation (6):

$$\boldsymbol{q_{x+t}^{i} = RR_0^{\left(\frac{\alpha - x - t}{\alpha - x}\right)} \cdot q_{x+t}^{s}}. \tag{6}$$

The mortality rate for the substandard individual can be expressed in terms of the standard mortality rate at age $x + t$, α, and RR_0. RR_0 is calculated to satisfy equation (1), using an iterative method such as Solver or Goal Seek in Excel.

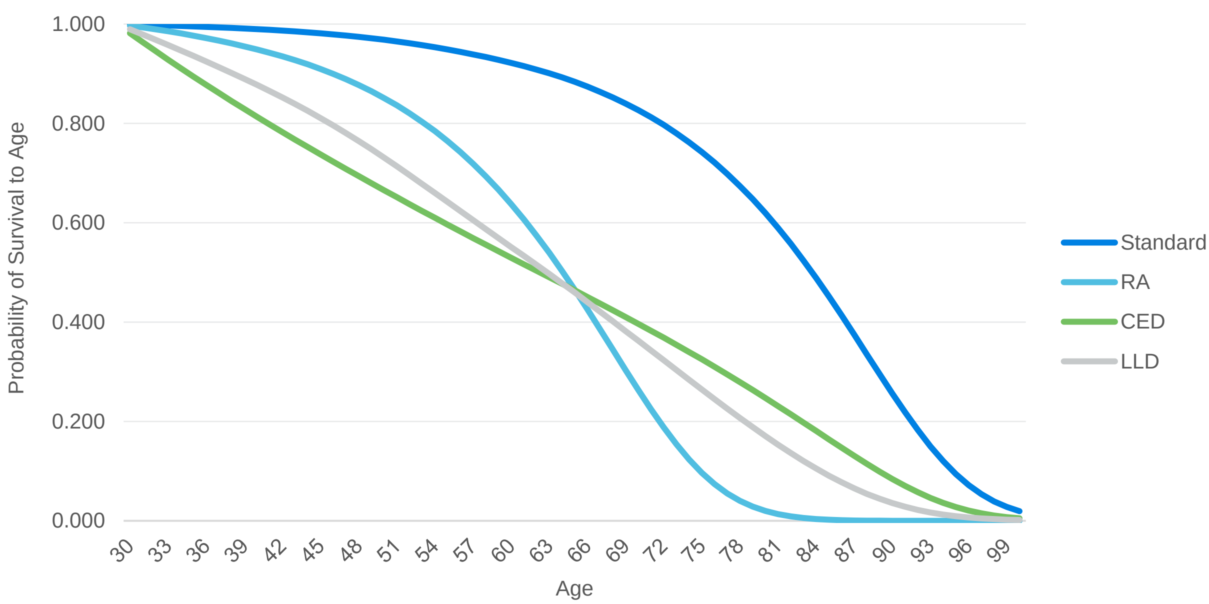
IV. Comparison of survival functions, life annuity factors, and mortality rates across basic methods

For pricing and valuation of annuities, the survival function, ${}_tp_x^i$, is more fundamental than the mortality rates, because the substandard immediate annuity factor, as shown in equation (7), is:

$$\boldsymbol{a_x^i = \sum_{t=1}^{\infty} v^t \ {}_tp_x^i}. \tag{7}$$

The question of SSSA mortality can be stated as, "How fast do we expect the annuitants to die, given their stated life expectancies?" Figure 1 shows how the answer varies across the basic methods, based on a sample annuitant who is a 30-year-old female with a rated age of 50. For comparison purposes, the standard (30-year-old female, no rate-up) is also shown. The same sample annuitant will be illustrated in Figure 2 through 5 as well.

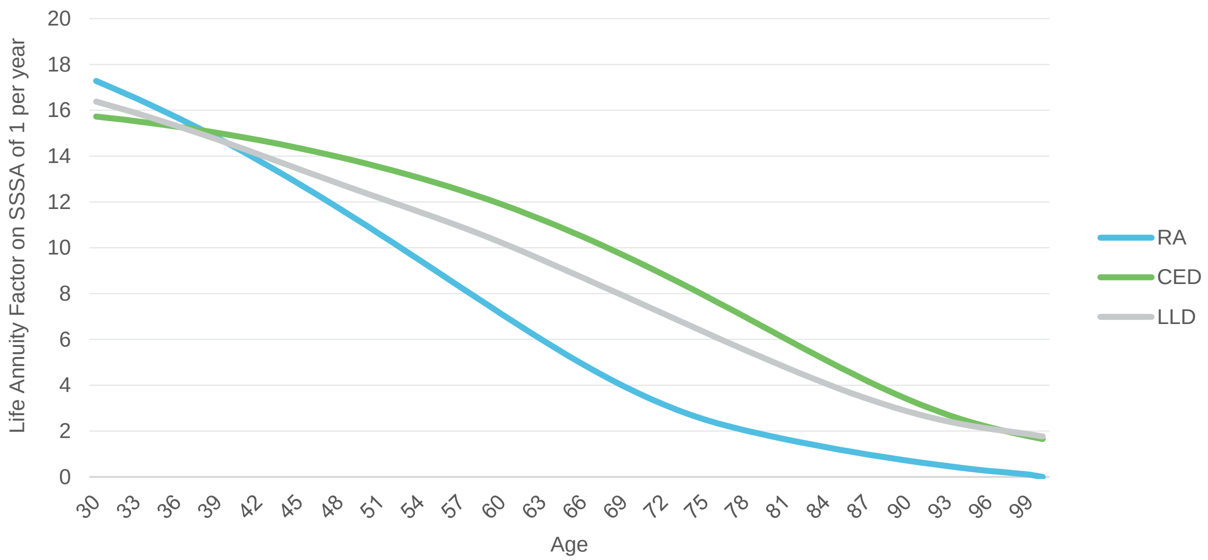
FIGURE 1: COMPARISON OF SURVIVAL FUNCTIONS – STANDARD AND 3 SUBSTANDARD METHODS



Based on the calculation, the RA method has much higher survivorship (lower mortality) in the early durations, offset by higher mortality in later durations. In contrast, the CED method has the opposite effect. The LLD method falls between these extremes.

Figure 2 shows the life annuity factor at 4% interest, a^{i}_{x+t}, for each of the basic methods on the sample annuitant.

FIGURE 2: COMPARISON OF SSSA LIFE ANNUITY FACTORS - 3 SUBSTANDARD METHODS



In a typical block with a mixture of ages, rate-ups, and durations, the CED method will often produce the most conservative results (highest life annuity factor), and the RA method will often produce the most aggressive results (lowest life annuity factor). For this reason, the CED method is the basis of U.S. statutory reserving.

The graphs in Figures 3, 4, and 5 show the mortality rates on the sample annuitant for each method by attained age. They are shown on three graphs, each for the same sample annuitant, differing only by age ranges.

FIGURE 3: COMPARISON OF MORTALITY RATES (30-50) – STANDARD AND 3 SUBSTANDARD METHODS

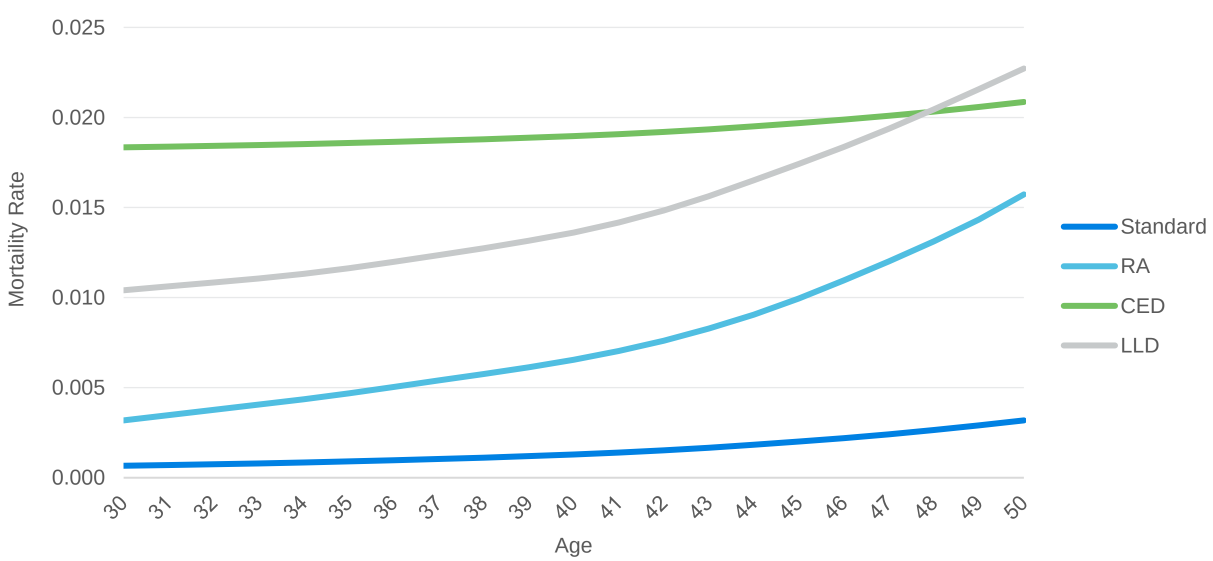


FIGURE 4: COMPARISON OF MORTALITY RATES (50-70) – STANDARD AND 3 SUBSTANDARD METHODS

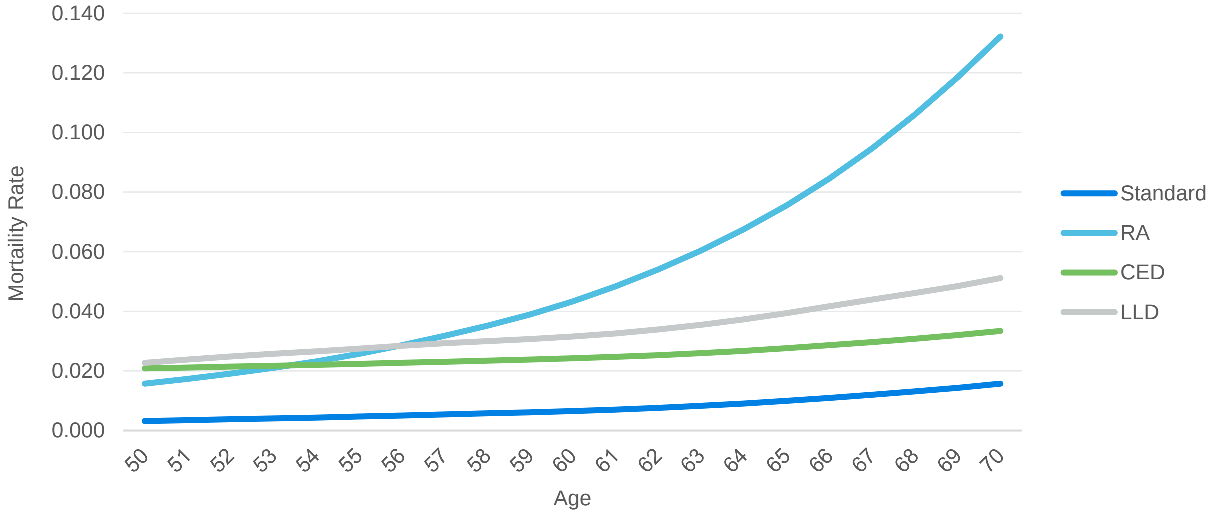
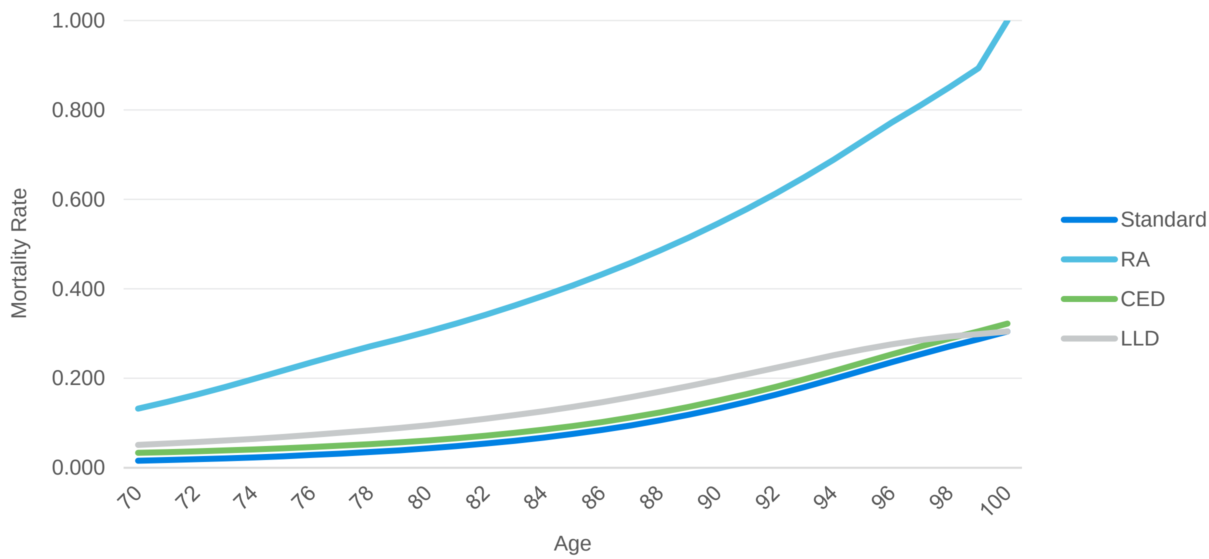


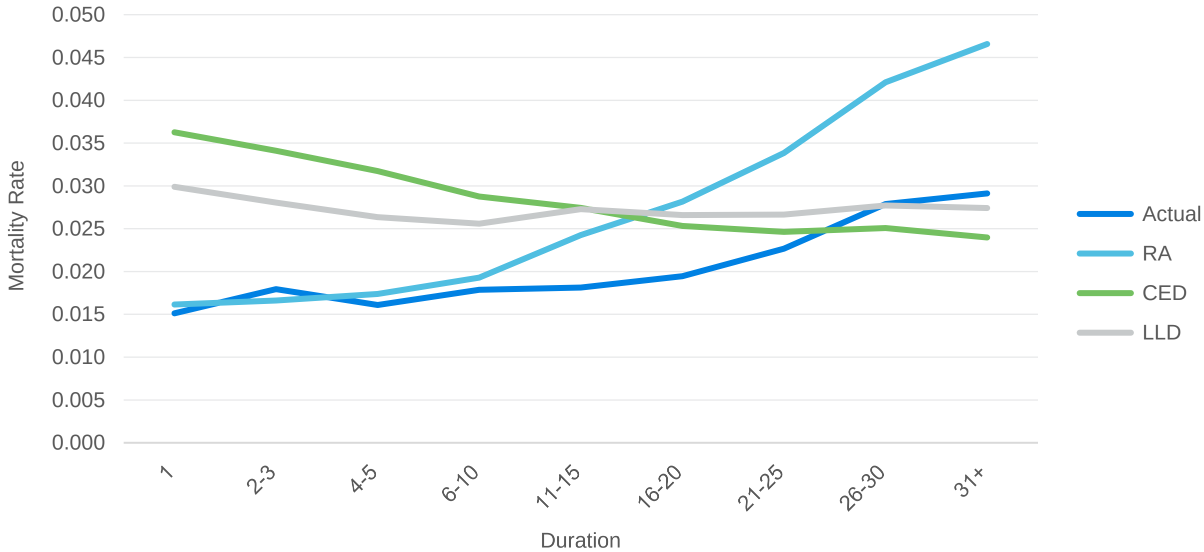
FIGURE 5: COMPARISON OF MORTALITY RATES (70-100) – STANDARD AND 3 SUBSTANDARD METHODS



V. Evaluating basic methods with actual policy data

In order to evaluate these three basic methods, we utilized actual policy level experience data from several blocks of SSSA, totaling approximately 350,000 exposures and 7,000 deaths. Figure 6 shows this comparison by policy duration.

FIGURE 6: MORTALITY BY DURATION – ACTUAL COMPARED TO 3 BASIC METHODS

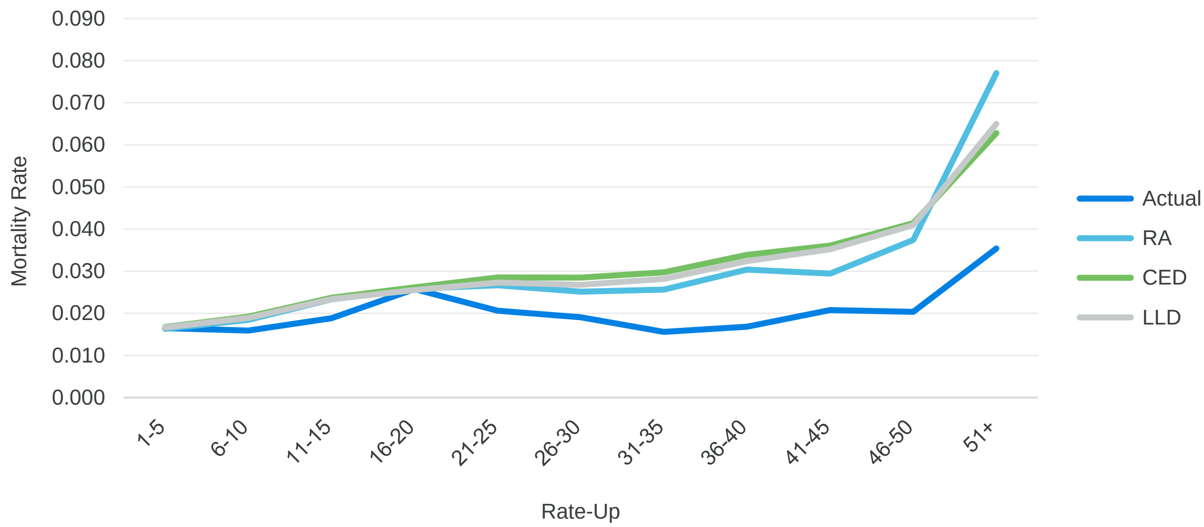


The main takeaway from Figure 6 is that the actual mortality lies below the expected mortality for most durations and methods. This is the result of a statistical bias likely arising during the underwriting process. It seems that, for most blocks of substandard structured settlements, the assigned rated age is too high for the actual impairment. This is specifically seen in policies with large rate-ups (20+ years), resulting in a high level of expected mortality under each of these basic methods.

As was the case in Figure 3, 4, and 5, mortality under the RA method is low in earlier durations but increases in higher durations. Conversely, the CED method has the highest mortality of the three basic methods in early durations, and the lowest in later durations. The LLD method falls in the middle.

We can see the upward bias more explicitly in a graph of mortality rates by the amount of rate-up, where rate-up is the difference between the rated age and the actual age, both at issue. Figure 7 shows mortality rates at rate-ups grouped quinquennially. Each of the methods overestimates mortality at all levels of rate-up, and the overstatement is an increasing function of rate-up beyond 20 years. This observation is the basis of one of the modifications that we introduce in Section VI below.

FIGURE 7: MORTALITY BY RATE-UP – ACTUAL COMPARED TO 3 BASIC METHODS



Note that each of the mortality rates in Figures 6 and 7 is comprised of a cumulation of thousands of policies, so we are not able to gain much insight from the average mortality rates or the slope of these averages. These patterns can be misleading as the groupings by duration or rate-up can have different average rate-ups, attained ages, etc.

VI. Overview of modified methods

In order to overcome the limitations of these basic methods, we modify them so that the ratio of actual deaths to expected deaths (A/E) is approximately 100%, and the expected mortality curve better resembles that of the actual mortality curve. A result of these modifications is that the Principle of Equivalent Life Expectancy no longer holds, as the modified methods are calibrated to fit actual experience data rather than only using information given in underwriting. The life expectancy may still be similar across the modified methods, but it will be different from what was assigned by the underwriter at issue.

We will discuss three modified methods, derived from each of the basic methods. They are called Modified Rated Age, Modified Constant Extra Deaths, and Modified Log-Linear Declining.

A. MODIFIED RATED AGE

This method modifies the basic Rated Age approach in two ways. First, it grades the impaired mortality back to standard mortality for advanced ages. This is accomplished by calculating a blended attained age based on the true attained age and the attained rated age during the grading period. By grading back to standard mortality, this modified method incorporates the improved mortality of individuals who have lived longer than what was assumed by the rated age at issue. The second modification is a set of two scalars, a standard scalar and a substandard scalar, each applied to the base mortality table and fit based on experience. This modification allows the method to better fit the overall A/E of a given block, while allowing for distinct standard and substandard mortality, beyond what the rate-up allows. Equation (8) shows the Modified Rated Age method:

$$q_{x+t}^{Mod\ RA} = \begin{cases} q_{x+r+t} \cdot SubstandardScalar & when \quad x+t \le GradeStart \\ q_{blended\ age} \cdot BlendedScalar & when \quad GradeStart < x+t \le GradeEnd \\ q_{x+t} \cdot StandardScalar & when \quad GradeEnd < x+t \end{cases}, \tag{8}$$

where the blended age grades linearly from the rated attained age to the true attained age over the grading period, and the blended scalar grades linearly from the substandard scalar to the standard scalar over the grading period.

B. MODIFIED CONSTANT EXTRA DEATHS

The Modified Constant Extra Deaths method makes two modifications to the additive constant, CED. First, a multiplicative scalar is applied, often reducing the constant to improve the fit to the data in earlier durations. The second modification introduces a factor referred to as the life expectancy ratio, which is the ratio of the life expectancy for the issue age of a policy to the life expectancy for the current attained age of the policy, both using standard mortality. This ratio is 1 at issue, and it increases with duration beyond that. These two modifications work together to decrease the amount added to standard mortality for early durations and increase it for later durations. Equation (9) shows the Modified Constant Extra Deaths method:

$$\boldsymbol{q_{x+t}^{Mod\ CED} = q_{x+t} + CED \cdot Scalar \cdot LifeExpectancyRatio}, \tag{9}$$

where the life expectancy ratio is given by equation (10):

$$\boldsymbol{Life\ Expectancy\ Ratio = \frac{e_x}{e_{x+t}}}. \tag{10}$$

C. MODIFIED LOG-LINEAR DECLINING

The final modified method is the Modified Log-Linear Declining method, which has three modifications. First, there is an adjustment to Relative Risk. A multiplier, constant across the block, is applied to the quantity $RR_t - 1$. This can increase or decrease the expected mortality rates and is used to improve the A/E for the entire block. The second modification is a reduction to the amount of rate-up for policies with high rate-ups, to correct the issue shown in Figure 7 above. The rate-up adjustment is 0 below some level of rate-up and decreases linearly above that. The final modification grades the impaired mortality back to standard mortality for older ages, for example over the range from 80 to 100, where data becomes sparse and actuarial judgment must take over. Equation (11) shows the Modified Log-Linear Declining method,

$$\boldsymbol{q_{x+t}^{Mod\ LLD} = \begin{cases} q_{x+t} \cdot RR_t^{Mod} & when \quad x+t \le GradeStart \\ q_{x+t} \cdot RR_t^{Mod} \cdot Factor + q_{x+t} \cdot (1 - Factor) & when \quad GradeStart < x+t \le GradeEnd \\ q_{x+t} & when \quad GradeEnd < x+t \end{cases}}, \tag{11}$$

where the factor represents a grading factor that grades off linearly from 1 to 0 during the grading period. RR_t^{Mod} is given by equation (12):

$$\boldsymbol{RR_t^{Mod} = 1 + Multiplier \cdot \left(RR_0^{\left(\frac{\alpha - x - t}{\alpha - x}\right)} - 1 \right)}, \tag{12}$$

and RR_0 is based on the adjusted rated age from equation (13):

$$\boldsymbol{AdjustedRatedAge = RatedAge - (RateUp - FixedRateUpLevel) \cdot Scalar}. \tag{13}$$

VII. Evaluating modified methods with actual policy data

In fitting a mortality assumption to experience study data, the goal is to get a good fit in-sample as well as in the future. However, it is difficult to tell whether a method has a proper fit well into the future without reviewing mortality experience after the assumption-setting period. In order to gauge how the modified mortality methods will likely perform on future data, we tested them by splitting the data into two different data sets, “training data,” the early years of experience, and “testing data,” the subsequent, later years. The parameters of the mortality methods were fit in the training period, and then tested *with those parameters* over the testing period. We looked at fit using A/E ratios, in aggregate, as well as across dimensions such as actual age, rated age, duration, and rate-up. This approach allows us to reduce the risk of “overfitting,” which is defined as creating a model that fits the training period very well and doesn’t fit in the test period. This exercise allows us to understand whether the fit models will be adequate in predicting future mortality. Of course, in an actual assumption-setting exercise, we would not split the data into training and test, because we will have used that tactic to ascertain that the best method produces good results out-of-sample.

Another important point to make prior to fitting to the data is that mortality by block is heterogeneous, meaning that mortality can differ substantially from block to block. This could be the result of differences in pricing era, marketing focus, mix of impairment types, level of underwriting bias, etc. We advocate for a common approach or method, but not for a common assumption to use on all blocks. Because of this, we must fit each of the blocks individually rather than as a large group when trying to determine estimated parameters.

The two graphs in Figures 8 and 9 show the expected mortality rates by duration and rate-up under each of the three methods compared to the actual mortality experience during the testing period. These graphs show the cumulative mortality across several blocks, but each block has parameters estimated from only its own training period. The graphs show the overall mortality rates. Duration or rate-up groupings without sufficient exposure or death count have been removed for simplicity.

FIGURE 8: MORTALITY BY DURATION – ACTUAL COMPARED TO 3 MODIFIED METHODS

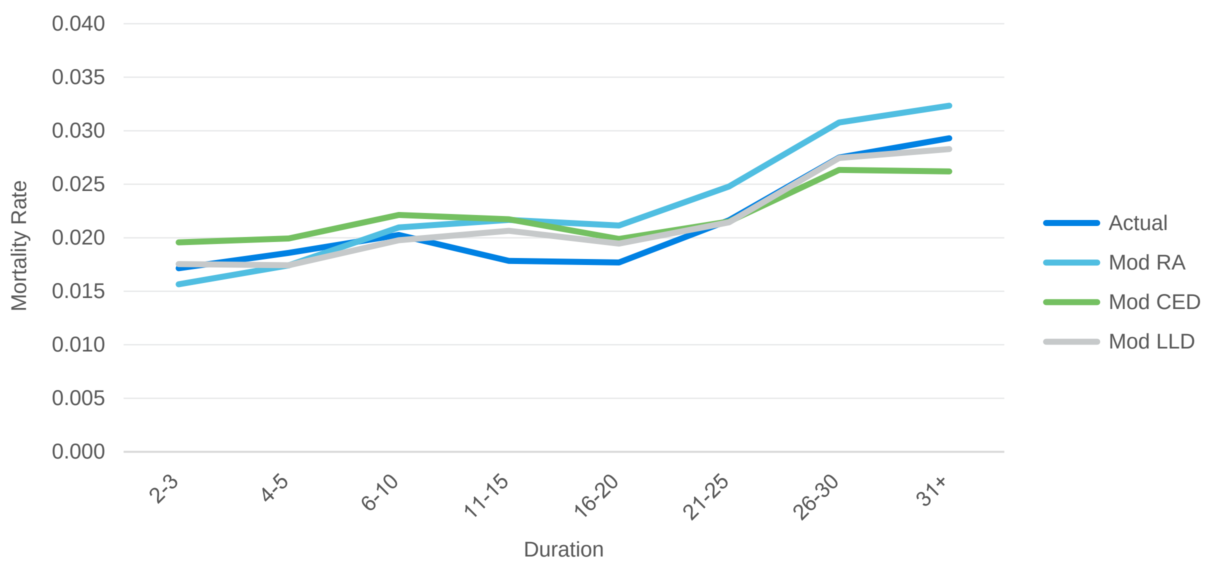
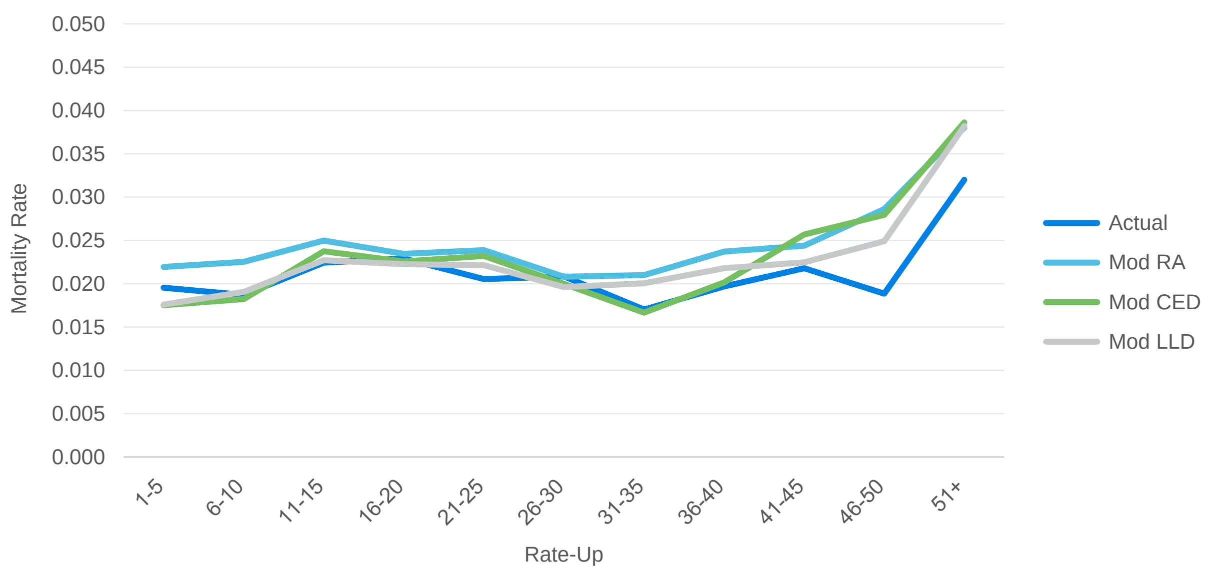


FIGURE 9: MORTALITY BY RATE-UP – ACTUAL COMPARED TO 3 MODIFIED METHODS



Here are some observations on the performance of the three modified methods, all of which point to the best method being the Modified Log-Linear Declining method.

1. At Durations 11+, the Modified LLD method is the closest to the actual mortality. Remember that this is an out-of-sample fit so the blocks, durations, impairment conditions, etc. led to the estimates of the parameters in the training period. Then Modified LLD with those parameters, on subsequent test data, performed quite well.
2. Likewise, at the highest rate-ups (41+), the Modified LLD method has the closest fit to the actual mortality. This is one of the key places where it is important to match mortality as well as possible, because policies with high rate-ups are often issued on juveniles with serious impairments, and the reserves are sensitive to the level of mortality over many years.
3. The Modified RA method understates mortality in early durations and overstates it in later durations. The Modified CED method overstates mortality in early durations and understates it in later durations. It would surely be possible to layer on more modifications to these methods, but it would likely result in overfitting, and it is not necessary when there is a method that provides a good fit already.
4. We do not show results at the block level to preserve anonymity. However, at the block level, the Modified LLD method had an overall A/E for every block of between 90% and 110%, a level of consistent accuracy that neither of the other methods achieved.
5. Also at the block level, Modified LLD had an A/E closer to 100% in a majority of the pair-wise competitions with the other two methods.

Another reason to favor the Modified LLD method is that each of the parameters has a specific purpose as it relates to fitting the actual mortality curve, allowing for easier explanation of parameters to decision-makers. First, the α parameter can be adjusted to manipulate the slope of the impairment wear-off in the mortality model to better fit the slope of mortality by duration. From there, the modification to reduce exaggerated rate-ups is used to fit the actual mortality by rate-up. Finally, the multiplier modification is applied to target a desired A/E of the entire block after already matching the slopes in Figures 8 and 9. To the extent that there isn't a lot of data, or the actuary has judgments to blend in with the data, this clear parameterization allows for judgment to be applied easily and transparently.

Conclusion

We examined the performance of three basic methods for modeling Substandard Structured Settlement Annuitant (SSSA) mortality and found them all lacking. We then looked at three modified methods that have been developed from those basic methods and found the Modified Log-Linear Declining method to be the best.

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[2] Strauss, D., Vachon, P., & Shavelle, R. (2005). Estimation of Future Mortality Rates and Life Expectancy in Chronic Medical Conditions. Journal of Insurance Medicine. Retrieved November 10, 2022, from https://www.lifeexpectancy.org/articles/ple.pdf.

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